

STUDENT ID NO							
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# **MULTIMEDIA UNIVERSITY**

# FINAL EXAMINATION

TRIMESTER 1, 2018/2019

### ETM7166 – DIGITAL SIGNAL PROCESSING SYSTEMS AND DESIGN IN TELECOMMUNICATIONS

(All sections / Groups)

23 OCTOBER 2018 10:00 AM - 1:00 PM (3 Hours)

#### INSTRUCTIONS TO STUDENTS

- 1. This Question paper consists of 11 pages with 4 Questions only.
- 2. Attempt ALL questions. All questions carry equal marks and the distribution of the marks for each question is given.
- 3. Please print all your answers in the Answer Booklet provided.

- (a) Digital signal processing algorithms have continued to find great use in increasingly wide application areas.
  - (i) With a block diagram, briefly explain the process of discrete-time digital processing of analogue signals.

[5 marks]

(ii) Briefly describe how does the spectrum of the sampled signal  $m_s(t)$  differ from the spectrum of the original signal m(t). Use an exemplary bandlimited signal and appropriate figures to illustrate the spectrum of this signal if sampling is performed at the Nyquist rate.

[5 marks]

(iii) Assume that the following analog signal  $x_c(t)$  is sampled at 4 kHz to generate the discrete-time sequence x[n].

$$x_c(t) = 3\cos(400\pi t) + 5\sin(1200\pi t) + 6\cos(4400\pi t) + 2\sin(5200\pi t)$$

Does the "aliasing" effect exist in the signal x[n]? Justify your answer.

[2 marks]

(b) The output y[n] of a digital filter is related to the input signal x[n] in the following equation:

$$y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

(i) What does the filter do?

[2 marks]

(ii) Is this a bounded-input bounded-output stable system? Justify your answer.

[2 marks]

(iii) Verify whether the system is causal and linear.

[4 marks]

(c) A linear time-invariant system has an impulse response of  $h[n] = -\delta[n+1] + \delta[n] - 2\delta[n-1] + \delta[n-1]$ . Calculate the filter output y[n] if the input sequence is  $x[n] = -2\delta[n+1] + \delta[n] + 3\delta[n-1]$ .

[5 marks]

(a) (i) Describe with a block diagram, the principle and procedure of implementing linear convolution using the discrete Fourier transform (DFT).

[6 marks]

(ii) Figure Q2 shows a practical approach to performing spectral analysis on a continuous-time signal g(t) by computing the discrete-time Fourier transform (DTFT),  $G(e^{j(0)})$  of its discrete-time equivalent g[n]. The DTFT is usually approximated by the DFT operation. In this context, explain the difference between high-density spectrum and high-resolution spectrum.

[4 marks]

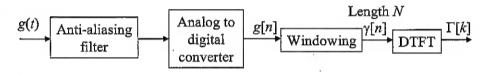


Figure Q2

(b) (i) A finite impulse response (FIR) filter can be designed to best approximate an ideal filter by simply truncating the infinite-length impulse response of the ideal filter. However, the frequency response,  $H_d(e^{i\omega})$  of the FIR filter obtained exhibits oscillatory behaviour. Explain the cause of this phenomenon.

[4 marks]

(ii) Suppose that an analog audio signal  $x_a(t)$  is contaminated by high-frequency noise. It is necessary to extract the clean signal using a low-pass filter with a cutoff frequency  $f_c = 5$  kHz, a transition width  $f_{\text{transition}} = 350$  Hz, and a stopband attenuation of 60 dB. Design the filter using a Kaiser window to meet the analog filter specifications with a sampling frequency  $f_s = 15$  kHz.

(Note: It is not necessary to determine the Bessel function explicitly for the answer.)

[6 marks]

(c) Obtain the system transfer function H(z) by designing a second-order lowpass Butterworth digital filter with a cutoff frequency (3dB point) at  $\omega_c = 0.2\pi$  using bilinear transformation with sampling interval  $T_s = 2$ . You may refer to the Appendix for the formula.

[5 marks]

(a) Explain how adaptive filters are different from the classical FIR and IIR filters.

[3 marks]

(b) Figure Q3 below shows the general structure of an adaptive signal processing.

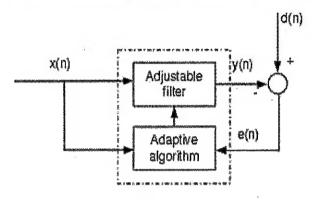


Figure Q3

(i) Describe the signals x[n] and d[n] and their relation.

[3 marks]

(ii) The least mean square (LMS) algorithm is an iterative solution to approach the minimum mean square error performance in adaptive signal processing. Write down the mathematical equations of the LMS algorithm for error calculation, output formulation and weight update, and discuss the factors that affect the convergence rate of the LMS algorithm.

[4 marks]

(iv) To improve the convergence speed and reduce the excess error, the normalized least mean square (NLMS) adaptive algorithm has been proposed by modifying the filter adaptation. Discuss the modification made, and how it helps in improving the convergence speed and reducing the excess error of the filter.

[3 marks]

(c) (i) One of the main problems associated with telephone communications is the line echo. Briefly explain the line echo problem.

[4 marks]

(ii) To reduce line echo, an adaptive echo canceller can be used. With the aid of appropriate diagram, briefly explain its basic principle.

[4 marks]

(d) In communication system, the transmitted data is distorted by the communication channel in different ways, with the most serious distortion being the inter-symbol interference. Explain how adaptive filtering can be used to solve inter-symbol interference.

[4 marks]

(a) Figure Q4 shows the block diagram of a Linear Prediction Coding (LPC) encoder.

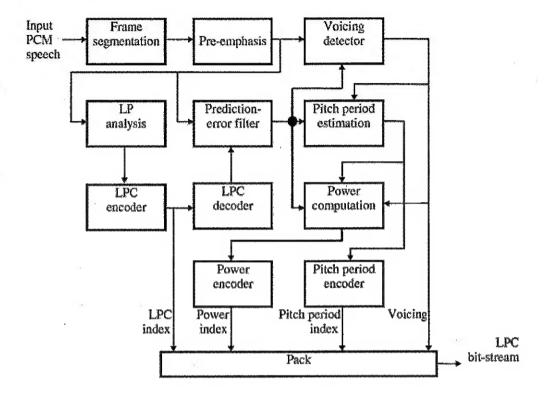


Figure Q4

(i) Briefly explain the four parameters (voicing bit, pitch period index, power index, and LPC index) that form the output bit-stream of the LPC coder.

[6 marks]

(ii) With the aid of a diagram, explain how the bit-streams are decoded into a speech by the LPC decoder.

[9 marks]

(b) A 256-sample voiced segment, with 16 bits/sample, is to be coded using parametric coding. The linear prediction filter employed is of 8<sup>th</sup> order, with each coefficient represented by 4 bits. The voicing, pitch period and scale factor are 1 bit, 4 bits and 5 bits respectively. Determine the compression ratio achieved by this parametric coder, and prove that it can achieve very high compression.

[4 marks]

(c) Discuss three fundamental limitations of the LPC speech coding model.

[6 marks]

## **Appendix: Formula Sheet**

### The z-transform

Properties of the z-transform

Property	x[n]	X(z)	$\mathcal{R}_x$
Linearity	ax[n] + by[n]	aX(z) + bY(z)	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	x[n-m]	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	x[-n]	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	x[n]*y[n]	X(z)Y(z)	$\mathcal{R}_x \cap \mathcal{R}_y$

### Common z-transform pairs

x[n]	X(z)	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n-n_0]$	$z^{-n_0}$	Possibly $\forall z$
u[n]	$\frac{1}{1-z^{-1}}$	z  > 1
-u[-n-1]	$\frac{1}{1-z^{-1}}$	z  < 1
$a^nu[n]$	$\frac{1}{1-az^{-1}}$	z  >  a
$-a^nu[-n-1]$	$\frac{1}{1-az^{-1}}$	z  <  a
$na^nu[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	z  >  a
$-na^nu[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	z  <  a

### Closed-form Expression for Some useful Series

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a} 
\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2} 
\sum_{n=0}^{N-1} n = \frac{1}{2} N(N-1) 
\sum_{n=0}^{N-1} a^n = \frac{1}{1-a} |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} |a| < 1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6} N(N-1)(2N-1) 
\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

### FIR Filter Design

Ideal bandpass

$$h[n] = \frac{w_2}{\pi} \operatorname{sinc}\left(\frac{w_2(n-M/2)}{\pi}\right) - \frac{w_1}{\pi} \operatorname{sinc}\left(\frac{w_1(n-M/2)}{\pi}\right),$$

$$n = 0, 1, \dots, M$$

Fixed windows

Window	Window function
Rectangular	w[n] = 1
Hann	$w[n] = 0.5 - 0.5\cos(2\pi n/M)$
Hamming	$w[n] = 0.54 - 0.46\cos(2\pi n/M)$
Blackman	$w[n] = 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M)$

Window	Passband ripple $20\log_{10}\delta_{p}$	Stopband attenuation $20\log_{10}\delta_s$	Transition width $ w_p - w_s $
Rectangular	-13	-21	$1.8\pi/M$
Hann	-31	-44	$6.2\pi/M$
Hamming	-41	-53	$6.6\pi/M$
Blackman	-57	-74	$11\pi/M$

Kaiser window

$$w[n] = \frac{I_0 \left(\beta (1 - (n/\alpha - 1)^2)^{0.5}\right)}{I_0(\beta)}$$

$$\beta = \begin{cases} 0.1102(A - 8.7), & A > 50\\ 0.5842(A - 21)^{0.4} + 0.07886(A - 21), & 21 \le A \le 50\\ 0, & A < 21 \end{cases}$$

$$M = \begin{cases} (A - 7.95)/(2.285\Delta w), & A \ge 21\\ 5.655/\Delta w, & A < 21 \end{cases}$$

Optimal Filter Design (Parks-McClellan Algorithm) Estimated Filter Order

$$N = \frac{-20\log\sqrt{\delta_p\delta_s} - 13}{14.6\Delta f}$$

### CLASSIFICATION OF LINEAR-PHASE FIR SYSTEMS

h[n] symmetric: $h[n] = h[N-n]$	h[n] antisymmetric: $h[n] = -h[N-n]$
Type I Linear Phase Filter	Type III Linear Phase Filter
$H(e^{j\omega}) = e^{-jN\omega/2} \sum_{k=0}^{N/2} a[k] \cos(k\omega)$	$H(e^{j\omega}) = je^{-jN\omega/2} \sum_{k=1}^{N/2} c[k] \sin(k\omega)$
a[0] = h[N/2]	c[k] = 2h[(N/2) - k]
$a[\kappa] = 2n[(N/2) - \kappa]$	
Type II Linear Phase Filter	Type IV Linear Phase Filter
$H(e^{j\omega}) = e^{-jN\omega/2} \sum_{k=1}^{(N+1)/2} b[k] \cos((k-1/2)\omega)$	$H(e^{j\omega}) = je^{-jN\omega/2} \sum_{k=1}^{(N+1)/2} d[k] \sin((k-1/2)\omega)$
$b[k] = 2h\left[\frac{(N+1)}{2} - k\right]$	$d[k] = 2h\left[\frac{(N+1)}{2} - k\right]$
•	$H(e^{j\omega}) = e^{-jN\omega/2} \sum_{k=0}^{N/2} a[k] \cos(k\omega)$ $a[0] = h[N/2]$ $a[k] = 2h[(N/2) - k]$ Type II Linear Phase Filter $H(e^{j\omega}) = e^{-jN\omega/2} \sum_{k=1}^{(N+1)/2} b[k] \cos((k-1/2)\omega)$

# IIR Filter Design

Normalized Butterworth lowpass

N	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$
1	1.000					- P		
2	1.414	1.000						
3	2.000	2.000	1.000					
4	2.613	3.414	2.613	1.000				
5	3.236	5.236	5.236	3.236	1.000			
6	3.864	7.464	9.142	7.464	3.864	1.000		
7	4.494	10.10	14.59	14.59	10.10	4.494	1.000	
8	5.126	13.14	21.85	25.69	21.85	13.14	5.126	1.000

Filter order

$$d = \left(\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}\right)^{0.5}$$
 
$$k = \frac{\Omega_p}{\Omega_s}$$

Design	Filter order
Butterworth	$N \ge \frac{\log d}{\log k}$
Chebyshev I, II	$N \ge \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ $N \ge \frac{\log(16/d^2)}{\log(2/u)}$
Elliptic	$N \ge \frac{\log(16/d^2)}{\log(2/u)}$
	$u = \frac{1 - (1 - k^2)^{0.25}}{1 + (1 - k^2)^{0.25}}$
	$\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$

### Frequency transformations

Target class	Transformation	Edge frequencies of target class
Highpass	$s  ightarrow rac{\Omega_p \Omega_p'}{s}$	$\Omega_p'$
Bandpass	$s  o \Omega_p rac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$	$\Omega_l,\Omega_u$
Bandstop	$s  o \Omega_p  rac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$	$\Omega_l,\Omega_u$

Impulse invariance transformation

$$H_a(s) = \sum_{k=0}^{p-1} \frac{A_k}{s - s_k} \longrightarrow H(z) = \sum_{k=0}^{p-1} \frac{T_s A_k}{1 - e^{s_k T_s} z^{-1}}$$

Bilinear transformation

$$H(z) = H_a(s) \Big|_{s = \frac{2}{T_s} \frac{1-z^{-1}}{1+z^{-1}}}$$
  
 $\Omega = 2 \tan(w/2)/T_s$ 

# Discrete-time Fourier Analysis

The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1$$

Properties of the DFT

Property	x[n]	X[k]
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1 X_1[k] + A_2 X_2[k]$
Time shifting	$x[\langle n-n_0  angle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k-k_0  angle_N]$
Time reversal	$x[\langle -n \rangle_N]$	$X[\langle -k \rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k \rangle_N]$
Convolution	$x[n]\circledast y[n]$	X[k]Y[k]
${\bf Modulation}$	Nx[n]y[n]	$X[k] \circledast Y[k]$